Wave Propagation in a Periodic Microstrip Line on a Multilayered Anisotropic Substrate

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***Abstract*— An efficient spectral domain full-wave algorithm based on 4 × 4 transfer matrix and integral equation method is developed for the periodic microstrip line on the multilayered anisotropic substrate. The substrate may consists of arbitrary number of layers whose permittivity and permeability are tensors of general form. It is shown, that in some cases, the quasistatic approach for calculations of characteristics of the dominant mode may be invalid even at very small frequencies. The method may be used for simulation of integrated circuits, which include anisotropic layers.**

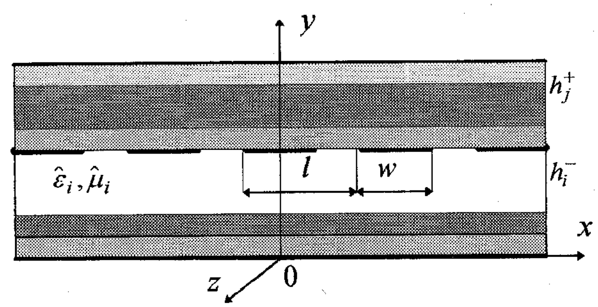

Fig. 1. Periodic microstrip line.

I. Introduction

TECHNOLOGICAL advances in microwave and millimeter-wave integrated circuits (MTC's) and microwave monolitic integrated circuits (MMIC's) have caused growing interest in multilayered planar transmission lines, which include the layers of anisotropic and gyrotropic materials [1]. Hence, some efforts have been devoted to the analysis of microstrip, slot, and coplanar lines on the substrates characterized by $\hat{\varepsilon}$ and $\hat{\mu}$ tensors (see, e.g., [2]–[4]).

A periodic microstrip line (PML) can be used as the design element of a number of control devices of microwave band: delay lines, phase shifters, slow-wave structures of vacuum electron devices [5], transducers for surface acoustic waves [6], electroacoustical transducers for such devices as acoustooptical Bragg cells, etc. The calculation problems of such devices can be solved in two steps. In the first step, one can calculate the parameters of PML (wave number and wave impedance) of dominant mode, as a function of frequency ω and the sizes and phase shift ψ of the wave field per period of PML. In the second step, using these parameters with the boundary conditions at the ends of the segments of PML and applying the methods of circuits theory, one can calculate the characteristics of these devices [7].

In other words, the wave number of the dominant mode of PML is a surface over the (ω, ψ) plane. Imposing the boundary conditions at the ends of PML segments, we cut the dispersion curve $\omega(\psi)$ of the studied periodic structure from the dispersion surface of PML $\gamma(\omega, \psi)$, where γ is the propagation constant of PML dominant mode.

The present work is devoted to the full-wave analysis of PML at the substrate, which consists of arbitrary number of layers whose permittivity and permiability are the tensors of general form.

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II. Configuration Under Study and Algorithm Description

The investigated structure shown in Fig. 1 represents an array of strip lines that is regular along the z axis and periodic along x axis with period l, w being the width of strips. h_j^+ and h_i^- denote the thickness of the jth layer above the array and the ith layer below the array, respectively. The strip lines are placed in the shielded multilayered anisotropic medium. Between the lower shield and the strip lines there are N_1 layers and between the strip lines and the upper shield there are N_2 layers. The thickness of conductors is assumed to be negligible and the dielectric loss is absent.

For the representation of field components in the layered structure we use the transfer matrix method [8]. We assume, the electromagnetic field in the periodic line to be varying as $e^{j(\omega t - \gamma z)}$. Let us denote the vector of tangential components of electromagnetic field of ith layer as $\boldsymbol{X}^{(i)} = (E_x, E_z, H_x, H_z)^T$, where T is the transposition symbol. Using the Fourier expansion of $\boldsymbol{X}^{(i)}$ in the spatial harmonics

$$\boldsymbol{X}^{(i)}(x, y, z) = e^{-j\gamma z} \sum_{n=-\infty}^{\infty} \boldsymbol{X}_n^{(i)}(y) e^{-j\beta_n x}$$

where $\beta_n = (\psi + 2\pi n)/l$, we can assume $\partial/\partial x = -j\beta_n$ and exclude E_y, H_y from the Maxwell equations. As a result, for each nth spatial harmonic we obtain the linear set of four ordinary differential equations (ODE)

$$\frac{d}{dy}\boldsymbol{X}_n^{(i)} = [A_n^{(i)}]\boldsymbol{X}_n^{(i)}. \quad (1)$$

Solving of a Cauchy problem for (1) may be written as

$$\boldsymbol{X}_n^{(i)}(h_i) = \exp([A_n^{(i)}]h_i)\boldsymbol{X}_n^{(i)}(0)$$

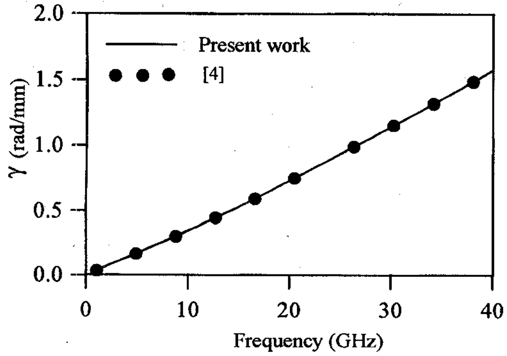

Fig. 2. Comparison with the results of [4]: propagation constant of the dominant mode with $\psi = \pi$ versus frequency. $N_1 = N_2 = 1, l = 4.318$ mm, $w = 0.5$ mm, $h_1^- = 0.5$ mm, $h_1^+ = 4.834$ mm, $\varepsilon_{xx} = 2.35, \varepsilon_{yy} = 2.0, \varepsilon_{zz} = 3.5, \mu_{xx} = 2.25, \mu_{yy} = 2.75, \mu_{zz} = 5.0$.

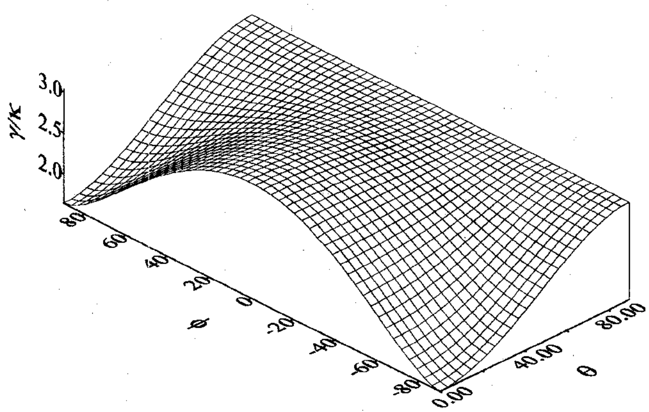

Fig. 3. The surface γ/k for phase shift per a period of the structure $\psi = 0$.

where $[M_n^{(i)}] = \exp([A_n^{(i)}]h_i)$ is the transfer matrix of the ith layer for the nth spatial harmonic. Using the transfer matrices of layers, we express the field components in the structure via the current density on the strips and reduce the boundary problem to the homogeneous integral vector Fredholm equation of impedance type

$$\int_{-w/2}^{w/2} \hat{G}(x, x')\boldsymbol{I}(x') = 0, \qquad |x| \leq w/2$$

where vector $\boldsymbol{I}(x')$ is the current density in the strip. The integral equation is solved using the Galerkin method. The weighted Chebyshev polinomials are used as basis functions.

III. Numerical Results

First, we present the results of testing the dependence of the algorithm convergence upon the maximal number M of the space harmonics taken into account (Table I) and upon the order of Galerkin's matrix K. The parameters of the structure are $N_1 = N_2 = 1, w/l = 0.5, h_1^-/l = 0.5, h_1^+/l = 0.05, \psi = 0, \lambda/l = 150$. The layers below and above the striplines are isotropic dielectrics with $\varepsilon_1^- = 10.6, \varepsilon_1^+ = 1$. The appropriate choice of basis, which takes into account the current singularity at the edges of strips, results in a very good the convergence upon K. For $M = 100$ we have $\gamma/k = 3.1081$ with $K = 3$ and $\gamma/k = 3.1083$ with K, changing from 5 to 17.

TABLE I
Convergence Investigation

M	20	40	100	300
γ/k	3.1128	3.1092	3.1083	3.1075

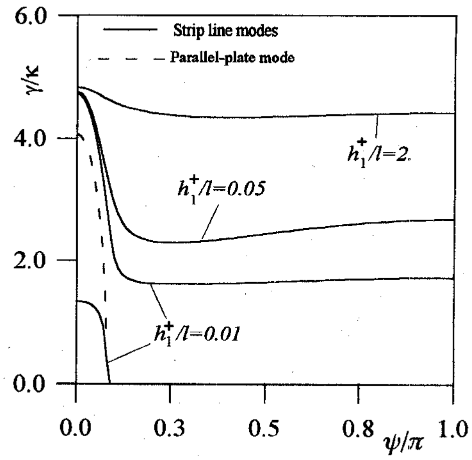

Fig. 4. γ/k of the PML dominant mode, parallel-plate dominant mode, and high-order mode of PML with $N_1 = 2; N_2 = 1; w/l = 0.5; h_1^-/l = 0.01; h_2^-/l = 0.5; \phi = 45°$.

Next, the proposed algorithm is verified by comparing of the results with [4] (see Fig. 2). PML for the case $\psi = \pi$ corresponds to a shielded stripline with the electric walls, placed in the $x = \pm l/2$ plane. The substrate is characterized by the diagonal tensors $\hat{\varepsilon}$ and $\hat{\mu}$ (see Fig. 9 of [4]). The layer above the stripline is vacuum.

Below we present the computation results for a shielded periodic microstrip line on the two-layered substrate, involving crystal $LiNiO_3$ (second layer). It has a strong anisotropy in the microwave band and approximate may be considered as unaxial crystal. For the first layers under and above the periodic strip line the relative permittivity is scalar end equal to one, as well as permeability. The surface of slow-wave factor γ/k versus polar θ and asimutal ϕ angles of anisotropy axis direction is shown in Fig. 3. Here θ is the angle between the layers plane and the anisotropy axis and ϕ is the angle between z-axis and anisotropy axis. Thus, the crystal layer is characterized by the nondiagonal tensor $\hat{\varepsilon}$. Arbitrary oriented crystal-based microstrip lines may be attractive for superconductive circuits [3], [9]. The parameters of the structure are $N_1 = 2; N_2 = 1; w/l = 0.5; h_1^-/l = 0.5; h_2^-/l = 0.25; h_1^+/l = 2; \lambda/l = 3; \varepsilon_\perp = 44; \varepsilon_\parallel = 29$ where λ is the wavelength, h_1^-, h_2^- and h_1^+ are the slab thickness below and above the strip lines and $\varepsilon_\perp$ and $\varepsilon_\parallel$ are the components of the permittivity tensor of crystal across and along the axis of anisotropy.

In Fig. 4 the slow-wave factor γ/k of the dominant mode for the different distances between strip lines and upper shield h_1^+ versus phase shift ψ is shown. The calculations were carried out with the wavelength $\lambda/l = 100$. The anisotropy axis lies in the (x, z) plane. It is significant that in the cases when ψ is small and the upper shield is closer to the strips than a lower shield, the dominant parallel-plate mode affects the characteristics of the dominant PML mode. This influence grows with the decrease of h_1^+. It is obvious that the abnormal behavior of the slow-wave factor with small ψ cannot be obtained using quasistatic approach even for very small frequencies, because it does not take into account the parallel-plate mode, which has no low-frequency cutoff at $\psi = 0$.

We also present in the Fig. 4 the slow-wave factor of the parallel-plate mode and the high-order mode of PML for the case of $h_1^+/l = 0.01$. This mode is the parallel-plate mode, perturbed by strip lines. It also has no low-frequency cutoff at $\psi = 0$. It has the field distribution similar to that of the leaky mode of a single strip line with a small air gap, which was described recently in [10].

IV. Conclusion

A transfer matrix and spectral domain formulation for solving the problems of wave propagation in the periodic microstrip line on the multilayered anisotropic substrate is described. Integral equations method leads to the matrix problem of small dimension M. $M = 3$–5 is enough for the calculation of the slow-wave factor of the dominant mode.

It is shown that quasistatic approach causes the incorrect results even if wavelength is large for the case of small ψ and when the air gap between the strips and the upper shield is small. Unusual behavior of γ/k for this case is described. Numerical results that illustrate the effect of anisotropy axis direction on the propagation characteristics are presented.

The created algorithm and the computer program may be used for the calculation of wave characteristics in the PML on the substrate, which can contain crystal layers, magnetized ferrites, and semiconductors, as well as piezoelectrics with arbitrary oriented axes of anisotropy and gyrotropy.

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